SEMESTRAL EXAMINATION
M. MATH I YEAR, II SEMESTER 2011-2012

COMPLEX ANALYSIS

The 7 questions carry a total of 110 marks. The maximum you can score is 100. Time limit: 3hrs

1. Find all entire functions f such that $f^3(z) = z \ \forall z \in \mathbb{C}$. [10]

2. If $f : \mathbb{C} \to \{z \in \mathbb{C} : \text{Re}(z) > 0\}$ is a function such that $g(z) = f(\bar{z})$ is an entire function show that f is a constant. [15]

3. If f maps the right-half plane into itself, f is holomorphic and vanishes at each positive integer show that $f(z) = 0 \ \forall z$. [20]

[Hint: use $1 - e^{-f}$]

4. If $u : \bar{U} \to \mathbb{R}$ is continuous and harmonic in U show that u can be expressed as the difference of two non-negative harmonic functions. [15]

5. Evaluate $\int\limits_0^\infty \frac{\cos(2x)}{(1+x^2)^2} dx$ by the method of residues. [20]

6. If f is an entire function such that $\lim\limits_{z\to\infty} \frac{f(z)}{z} = 0$ show that f is a constant. [15]

7. Construct a conformal equivalence between the first quadrant $\{z : \text{Re}(z) > 0 \text{ and } \text{Im}(z) > 0\}$ on to the upper-half of the unit disk $\{z \in U : \text{Im}(z) > 0\}$.[15]